7. Verify Mean Value Theorem of $f(x) = x^3 - 3x + 2$ for [-1, 2].

Solution:

here,

$f(x) = x^3 - 3x + 2$ for $x \in [1,-2]$.

i. $f(x) = x^3 - 3x + 2$ is an polynomial function and polynomial functions are continous on their domain. So, f(x) is continous $\forall\ x \in [-1,2]$.

ii. $f'(x) = 3x^2 - 3$ is defined $\forall\ x \in (-1,2)$.
So, f(x) is differentiable $\forall\ x \in (-1,2)$.

iii. Thus, f(x) satisfies both conditions of Mean Value Theorem so, by Mean Value Theorem there exists a point $c \in (-1, 2)$ such that,

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

or, $$3c^2 - 3 = \frac{f(2)-f(-1)}{2-(-1)}$$

or, $$3(c^2 - 1) = \frac{4-4}{3}$$

or, $$c^2 - 1 = 0$$

$$\therefore\ c = \pm 1 \in (-1, 2)$$

Hence, Mean Value Theorem is verified.